

Stevens Institute of Technology Plume-Surface Interactions for the Moon and Mars Phase 1 Report

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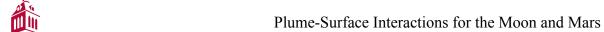
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"I pledge my honor that I have abided by the Stevens Honor System"

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Abstract

Plume surface interactions studies the environment that is created from rocket plumes. When a rocket is attempting to land on a surface such as the Moon or Mars, the exhaust interacts with the ground in a very violent manner by launching particles of all sizes as well as heating them up. The two biggest issues are the potential damage that can be caused to the ship as well as the lack of visibility caused by the dust. There have been a lot of CFD (computational fluid dynamics) models created to try to accurately portray this problem however there is little experimental data in similar conditions.

The goal of this project is to create an experimental setup that produces flow at similar speeds and conditions to that of the moon (~340m/s in a vacuum) as well as design a plate that will act as the surface of the moon. The plate will also be adjustable allowing for multiple "landing angles" to be tested. Using various sensors, data can be collected about the flow acting upon the plate. Schlieren imaging techniques will attempt to be employed to study the flow. CFD models will also be used to compare the experimental results to the computational ones. This will hopefully allow for accurate testing to be done without the need for wind tunnels.

Understanding the plume surface interaction will allow for safer landings for spacecraft as well as more complex landing sequences. Apart from space applications, this research can be used to safely land helicopters and planes into areas with a high volume of small particles such as deserts.

Introduction

1.1. Problem Statement

This project will provide a new way of understanding the interactions responsible for many of the issues current air and space craft experience during landing sequences. Because of the high importance of any data collected, the experimental setup designed needs high measurement accuracy and consistency. The design must be compatible with the existing high-speed shock tunnel's parameters and available space. While staying within the tunnel's allowed parameters, the apparatus should be adjustable in terms of height and plate angle. Pressure sensors will be used to record data and must be compatible with the apparatus as well as the shock tunnel. The setup must be able to withstand the large expected force loads due to the high-speed flow acting on the plate. This requires strength of any joints implemented as well as overall quality of materials. The setup should be able to collect a wide range of high-quality and consistent data. Adjustability within the setup, for example adjustable heights and angles, will contribute to this goal.



1.2. Business Case

The business case for this project is based around advancing the understanding of fluid and particle behavior during plume surface interactions in low pressure environments. Current computational fluid dynamics (CFD) struggle to simulate the conditions of these interactions. If accurate physical data can be generated by this project, it could be used to develop new algorithms and verify results from CFD simulations. By developing a physical model, the team would be providing data that is very valuable to the companies that develop these CFD programs.

Additionally, companies and organizations such as NASA or SpaceX have a vested interest in understanding plume surface interactions. The interactions that this project aims to model are based on landers or other craft descending onto the surface of a low pressure environment such as the Moon or Mars. Both NASA and SpaceX plan to launch missions to these destinations, and understanding plume surface interactions can take away some of the risk or unpredictability of their missions. These companies not only use CFD to test aspects of their missions but they may also be inclined to directly use the physical data developed from this project or advance the scope of the project further.

1.3. Project scope

The team is to improve upon the pre-existing sonic impingement design such that more quantitative measurements (i.e. the pressure at various points on the plate) can be taken. The team will also improve the design such that pre-existing variables such as the exact angle of the plate with respect to the nozzle and the exact distance between the altitude of the plate and the tip of the nozzle can be defined with much greater accuracy (\pm 1° and \pm 0.001m respectively). Implementing these changes will allow for a wider range of experimental situations to be tested and allow for a more quantitative comparison to modern computational fluid dynamics (CFD) simulations. Additional improvements not specified may also be made if the team believes them to increase the accuracy of measurement, allow for additional experimental tests, or otherwise contribute to the primary goal of the creation of reliable experimental data.

1.4. Literature review

A majority of the literature review consisted of the report regarding the pre-existing experimentation performed last year (Daigle et al., 2022). The concepts, assumptions, and equations present in this report were verified through the review of the referenced texts (Anderson, 2000; Anderson, 2021; Anderson 2022; Kuda, 2015).

Methodology

2.1. Conceptual designs

With this project being a continuation of the previous Senior Design group's work, the group was tasked with implementing certain improvements to the design, as well as ensuring other conditions were met within the design to effectively gather data. Regarding fabrication, the design of the plate must include a mount that can fit inside the wind tunnel test section,



specifically in front of the second window (Figure 1). In last year's project, the flat plate angle was adjusted manually, which resulted in the test section needing to be opened many times. This required the tunnel to be depressurized to create vacuum-like conditions multiple times, leading to long waiting times for testing (when compared proportionally to the sample time of the test, the waiting time was extremely long). This year's group wants to avoid that situation altogether by using motors and sensors to adjust the plate's height and angle remotely. There are two main avenues that the group has brainstormed about: actuating the plate, or actuating the nozzle itself. The following concept designs explore both of those options. Additionally, along with Schlieren imaging, another type of data collection that the group wants to explore is fluid pressure data, and so incorporating pressure sensors into the plate was given importance in the design process.

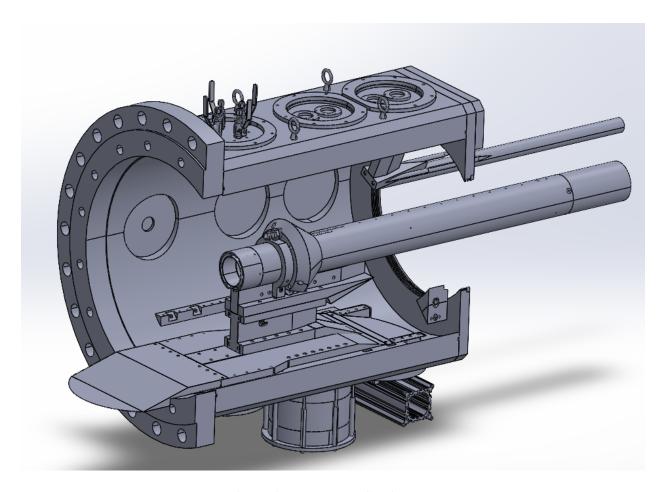


Figure 1: Tunnel Test Section



Concept #1: Linear Actuators

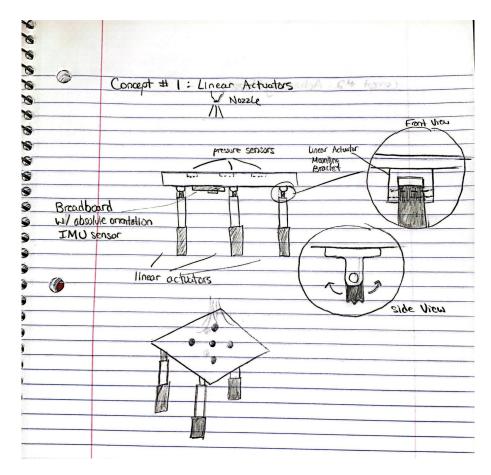


Figure 2: Concept Design 1

For this design, the group decided on using three linear actuators with hinge mounts on each end to effectively control both the height and the angle adjustment of the plate. Inputs from the three actuators would allow the group to create different angles, and an absolute orientation IMU sensor on the bottom side of the plate would allow the group to accurately measure what angle the plate is at. With the sensor and actuator wires running out of the bottom of the test section to an exterior board, the plate would be able to be fully manipulated without having to open up the test section. The nozzle would stay in a fixed position for all tests.



Concept #2: Actuating Nozzle

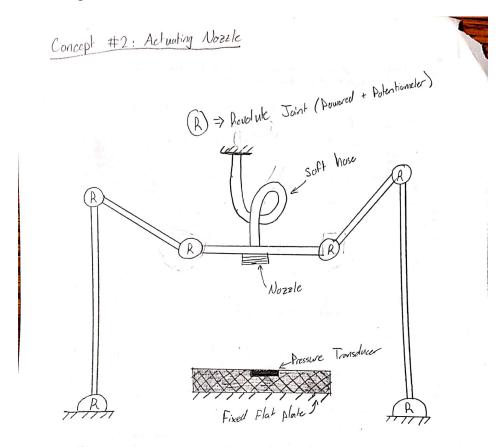


Figure 2: Concept Design 2

For Concept #2, the group focused on keeping the plate with the pressure sensors fixed and changing the orientation of the flow through an actuating nozzle. The angle of the nozzle would be adjusted through a linkage powered by stepper motors, which would receive inputs from potentiometers to set the angle and height. Extra tubing would be made available above the nozzle to allow it to effectively translate up and down.



Concept Design #3: Actuating Base

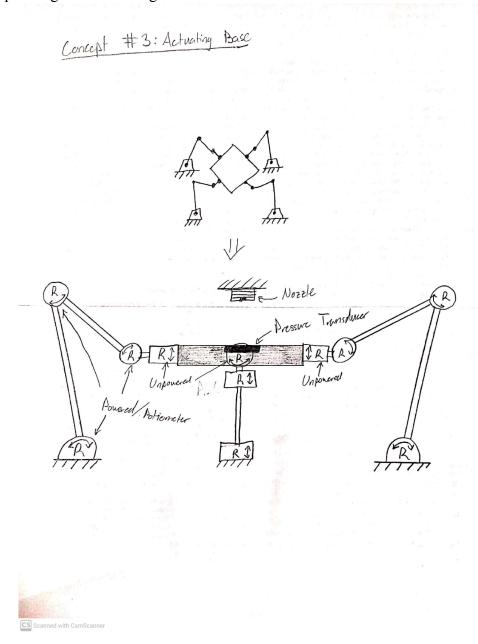


Figure 4: Concept Design 3

The third concept design takes a similar approach to Concept #2, however this time applying the stepper motor and potentiometer-powered linkage to actuate the base plate. A motor and arm would attach to each side of the plate, and a potentiometer at each servo would allow the group to read and monitor the angle and height adjustment at each motor.



		Α	В	С	D
		Previous Year	Linear Actuators	Actuating Nozzle	Actuating Base
Selection Criteria		Concept A	Concept B	Concept C	Concept D
Height Elevation		0	1	1	1
Angle Adjustment		0	1	1	1
Pressure Sensor Compatability		0	1	1	1
Remote Height/Angle Adjustment		0	1	1	1
Joint Stiffness		0	0	-1	-1
P	luses	0	4	4	4
Sa	ames	0	1	0	0
Mir	nuses	0	0	1	1
	Net	0	4	3	3
	Rank	3	1	2	2

Decision Matrix

In regards to ranking the individual concepts, the group developed a decision matrix, comparing each concept to Concept A, last year's project. A score of 1 was given if the concept theoretically would be better at the given specification than the project from last year, 0 if it would perform similarly, and -1 if it would perform worse.

2.2. Overview of technical approach

The technical approach will consist of analytical preliminary calculations, verified by simulated results. The analytical calculations will be carried out by hand and include conservative assumptions to both simplify the model and incorporate a factor of safety. Basic fluid simulations will be conducted on Ansys, and while the yielded results are expected to be accurate, the results will not be as conservative as the analytical calculations. However, both types of results will be useful in cross-verification, as well as independently influence design decisions.

2.2.1. Type of analysis, simulation, prototype

As previously stated, numerical calculations will be used in tandem with simulated results. The numerical calculations/analysis will be discussed in greater detail in section 2.7. The simulations will consist of choked flow impinging on a flat plate. The experimental studies previously conducted will be virtually replicated and simulated. Pressure at specific points along the plate will be probed to give an estimate for the forces and moments expected to be imparted on the flat plate. Screenshots of the simulation will be compared to the Schlieren images taken from the experimental results to verify the validity of the simulation. The angle of the plate with respect to the nozzle can be varied throughout the simulated trials to yield a better range of



expected forces/moments. The forces/moments present on the plate will be used in a later numerical analysis/simulation to determine gear ratios/stalling torques necessary to ensure the plate remains stationary when under the given load. This analysis will be reserved for only the final design since the analysis will be unique for each design choice.

2.3. Software/hardware/system/environment requirements

The environmental requirements for this project are a wind tunnel which can produce supersonic flow to be able to accurately represent the speed as well as a vacuum chamber to accurately represent the environment. For hardware requirements, basic machining tools will be used to alter the flap plate that will be used for testing. Lastly, for software a CFD simulation software will be used to be able to compare the experimental results to the theoretical ones.

2.4.Individual responsibilities

To effectively meet deadlines and progress at a steady rate, the team will divide the labor evenly between them. Uesli is currently in charge of the electrical, and programming components as well as cable management if needed. Michael and Maria are responsible for understanding and improving the nozzle design to reproduce the experimental results as well as work towards more accurately controlling the flow rate. Grant and Jett are responsible for engineering a means of actuating the flat plate/nozzle. Jett is responsible for notes during group and advisor meetings, and Grant is the designated primary contact for the project. While each member leads their own sub-project, all members are expected to contribute to all sub-projects when necessary.

2.5. Justification for solution choice

The reasons for choosing these concepts will be discussed in further detail later.

2.6. Component sizing and selection

For this project, the team is relying on using the Stevens Hypersonic Shock Tunnel in order to create the correct environmental conditions for the desired data. As a result, all component sizes and designs must be made with the dimensions of the wind tunnel's test section in mind. The team must ensure that their design can be placed inside of the test section while still maintaining the range of motion to adjust the height and angle of the flat plate. The team's design must also be compatible with the existing mounting surfaces within the wind tunnel.

In order to achieve this, the team was given access to SolidWorks files of the Stevens Hypersonic Shock Tunnel in order to find the optimal placement, size, and orientation of components in their design. This will also help to ensure that the final design fits properly in the tunnel without issues.

2.7. Analysis and calculations

To perform a preliminary analysis of the forces experienced on the plate, several assumptions and constraints must be made. Firstly, the analysis will include the nozzle oriented such that it is perpendicular to the flat plate. It will be assumed that the nozzle is choked, so it is only capable of producing Mach 1 speed flows. To estimate conservatively, it will be assumed that the flow will stagnate entirely when impinged by the plate. Additionally, it will be assumed



that the specific heat ratio varies negligibly across the expected temperature range. To further simplify calculations, it will be assumed that the test section will be held at vacuum. Finally, for the purpose of this force calculation, the fluid will be assumed inviscid.

From an analysis of the experimental data taken last year, it can be determined that the flow will expand upon leaving the nozzle. The flow expansion of the flow is a function of the distance between the nozzle and the flat plate. Schlieren images were analyzed and it was determined that the greatest area in which the jet first contacts the flat plate is approximately 0.868 inches in radius, experienced when the nozzle was 1.25 inches from the plate. Assuming radial symmetry, the area can be estimated to be 2.37 in².

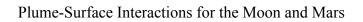
Utilizing equation 1, or more conveniently equation 2 (Appendix A), the stagnation pressure can be calculated for this area and subsequently, a conservative force estimate can be calculated. The forces and moments experienced on the links and joints respectively can be calculated to determine the feasibility of each actuating nozzle/plate design.

2.8. Engineering justification of the final design

The concepts chosen here are evaluated to meet the engineering design needs of this project and its ending goals. The use of linear actuators allows for a higher range of motion than the previous existing design used. The linear actuators are also rated with a high enough strength to withstand the forces acting on the plate due to the high speed flow. The use of electrical components allows for adjustability without opening the shock tunnel between each test. This allows for more testing to be performed at a time and less human error in setting up and adjusting the apparatus. Each design allows for the necessary pressure sensors to be used in order to collect data from the flow over the plate. The improved overall adjustability and strength of the concepts generated allow for more consistent and reliable data collection than the previous setup offered.

2.8.1. Summary of analysis

Many design concepts were discussed but three main concepts have been generated for further analysis and comparison. These concepts all work toward the goal of having an adjustable but consistent experimental setup in order to collect data to understand the plume surface interactions. Each concept was drawn out to understand and compare their basic details and functionality. Calculations will be done to further compare how each design will perform under their expected conditions under high forces in the vacuum of the shock tunnel. As discussed, modeling software will be used to create prototypes for analysis, as well as a simulation of the flow conditions they will be subjected to.





2.9.1. House of quality

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				Direction of Improvement:	V	X	V	Ā	Ă	X	X	
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				Quality Characteristics		Accuracy of Angle Measurement (Sensor Implementation)		ction				
	Row			(a.k.a. "Functional Requirements" or		ent (s		Compatability with Tunnel Test Section				
	Max Relationship Value in Row			"Hows")		nrem	ged	i ii			5	
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Row#	Мах	Rela	Wei	Requirements" or "Whats")	Cost	Acci	N I		Plate	Deg	Pres	
1	9	13.9	10.0	Height Elevation	Θ			Θ				
2	9	13.9	10.0	Angle Adjustment	Θ	Θ	Θ	Θ				
3	3	13.9	10.0	Pressure Sensor Compatability	A						0	
4	9	9.7	7.0	Remote Height/Angle Adjustment	0	Θ				A		
5	3	5.6	4.0	Higher Mach# Nozzle	A			0	0			
6	9	5.6	4.0	Improved Solonoid	Θ			A				
7	9	13.9	10.0	Joint Stiffness	0				Θ			
8	9	9.7	7.0	Range of Data Acquisition		Θ	A		Θ	0	Θ	
9	9	13.9	10.0	Quality of Data Acquisition		Θ			Θ		Θ	
10												_
				Target or Limit Value								
				raiget or Limit value								
				Difficulty								_
				(0=Easy to Accomplish, 10=Extremely Difficult) Max Relationship Value in Column	9	9	9	9	9	3	9	
				Weight / Importance	390.3	425.0	134.7	272.2	354.2	38.9	254.2	
				Relative Weight	20.9	22.7	7.2	14.6	18.9	2.1	13.6	

	Legend	
Θ	Strong Relationship	9
0	Moderate Relationship	3
	Weak Relationship	1
++	Strong Positive Correlation	
+	Positive Correlation	
	Negative Correlation	
	Strong Negative Correlation	
lacktriangle	Objective Is To Minimize	
	Objective Is To Maximize	
Χ	Objective Is To Hit Target	



2.9.2. **DFMEA**

DFMEA table							
System/ Function	Potential Failure Mode	Severity (1-10)	Potential Causes of Failure Mode	Current Preventitive Action	Current Detection Activities		
Flat Plate Assembly/ Serves as test surface for experimentation	Uncompatibility with sensors or tunnel test section	5	improper CAD medelling	Include Detailed part drawings and verify component measurements	Test fit components when possible		
Nozzle Assembly/ Provides sonic flow into plate surface	Component Failure Under Vaccuum	5	Incorrect choice of materials	Technical Analysis to determine required specifications	Preliminary testing		
Angle Adjustment System/ Electrical subsytem to alter the plate angle while the setup is within the shock tunnel	Inability to hold the plate in desired orientation	10	choice of components	Technical Analysis of the force exerted by the flow, Research into linear accuators and position	Preliminary testing		
Pressure Sensor Subsystem/ Data Acquisition System to measure pressure differentials across the flat plate	Inaccurate Pressure Measurements	8	placement within plate	Consideration of sensor placement	Preliminary calibration and data acquisition tests		

Table 1: DFMEA

The above DFMEA table above was generated to apply DFMEA to this project. Due to the experimental nature of this project rather than a traditional product to be manufactured, some aspects of DFMEA may be left out at this time. In this Table, severity was given a ranking from 1 to 10 based on how serious the team anticipates failure in that system to be.

In terms of the flat plate assembly, the team primarily needs to ensure that the components all fit together. This can easily be done by verifying measurements and design



dimensions before making the plate, and test fitting components whenever possible. Similarly, the design of the nozzle assembly will mostly rely on technical analysis and material selection to ensure that it can be used under the low pressure conditions within the shock tunnel.

The system for angle adjustment has been the primary focus of the team as they have developed concepts. This is the main design challenge that the team intends to tackle. The team will perform a technical analysis of the anticipated forces on the plate during a test. This information will be used to determine what components are available within the team's budget that are capable of providing angle adjustment for the different concepts and use it to help with design selection moving forward.

Finally, the team will need to incorporate pressure sensors into the design of the plate. This system needs to be tested on its own before it is implemented into the plate. This will allow the team to understand the behavior of the sensors to decide on their placement.

2.9.3. PFMEA

PFMEA table							
System/ Function	Potential Failure Mode	Severity (1-10)	Potential Causes of	Current Preventitive	Current Detection		
	Wiode	(1-10)	Failure Mode	Action	Activities		
Flat Plate Assembly/	Vibrations		Lack of	Include	Observe plate		
Serves as test surface for	during testing	8	damping in	Dampening	behavior during		
experimentation	damily costing		plate support	into design	tests		
Nozzle Assembly/ Provides sonic flow into plate surface	Failue to produce fully developed, Mach 1 flow	5	Impropper nozzle Design	Technical Analysis to determine Nozzle and flow specifications	Preliminary testing		
Angle Adjustment System/ Electrical subsytem to alter the plate angle while the setup is within the shock tunnel	Innaccurate Angle Measurements	10	Inability to maintain a consistent angle during test	Technical Analysis of the force exerted by the flow, Research into linear accuators and position	Implementation of 9-DOF Absolute Orientation IMU Fusion Breakout into design		
Pressure Sensor Subsystem/ Data Acquisition System to measure pressure differentials across the flat plate	Inaccurate Pressure Measurements	8	impropper calibration	Consideration of sensor placement	Preliminary calibration and data acquisition tests		

Table 2: PFMEA



The team also adapted the PFMEA format above to best fit this project. While these failure modes are failures that may occur during the experimental process, they actually need to be anticipated in the design of each system. For each of the systems in the PFMEA table, the team's goal is to maximize the validity of experimental data once the final design is ready for experimentation.

In the design of the flat plate assembly, the team will prioritize limiting vibrations. While analyzing the Schlieren data from the previous design team, it was noted that the plate vibrated during the test. In order to collect the most accurate data possible, the new plate design will keep dampening in mind.

The nozzle assembly does not pose a huge threat in terms of failure. The team's concerns are that it will fail to produce the proper flow, or that the structure will not withstand the vacuum. In order to prevent these failures, the team will primarily turn to technical analysis of the throat diameter required in the nozzle as well as the optimal flow rate.

The angle adjustment system poses the greatest challenge in terms of its anticipated severity during failure. The angle of the plate must be accurately measurable and consistent in order for the collected data to be useful. This system is also a primary target for improvement from the previous team's design. The design of the adjustment system relies heavily on the design of the plate assembly itself. The team intends to use technical analysis to anticipate the forces the system will encounter and determine further part specification requirements. The team also intends to include an orientation sensor into the design to display the plate's position in each test.

The pressure sensor system also poses a challenge for the team. The pressure sensors available in the lab, ICP Pressure sensor model 106B52, need to be tested and characterized before they are implemented into the final design. This testing will help determine the placement of the sensors within the plate. It will also allow the team to ensure that the data collected from the sensor will be relevant.



2.10. Bill of Materials

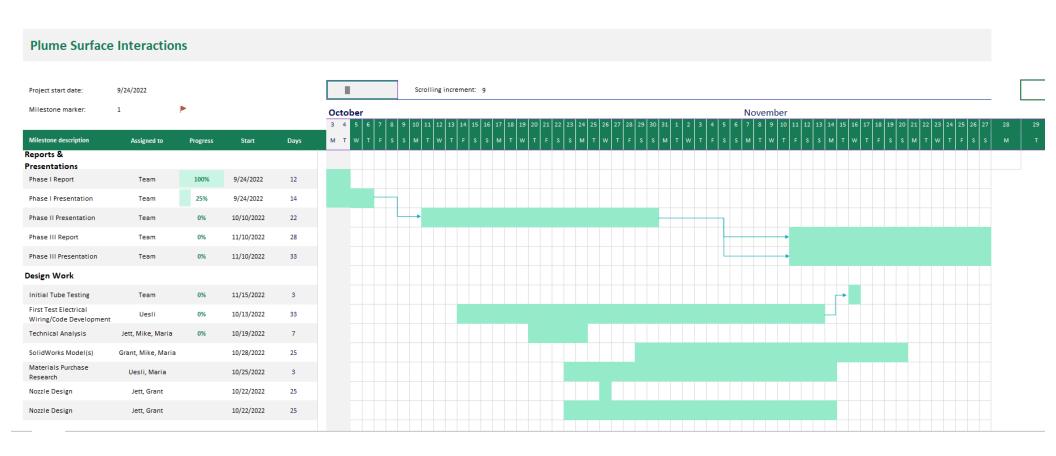
Bill of Materials								
Description	Part Number	Vendor	Quantity	Cost Per Item	Total	Notes		
Adafruit 9-DOF Absolute Orientation IMU Fusion Breakout	BNO055	Adafruit	1	\$34.95	\$34.95	Desired purchase for any concept design chosen		
Ball Joint	60645K61	McMASTER- CARR	3	\$9.75	\$29.25	Dependent on design chosen		
Linear Actuator	HLS12-100380-6V	Servocity	4	\$100.00	\$400.00	Dependent on design chosen		
Aluminum 6061	P61.25T651ND	BuyMetal	1	\$38.37	\$38.37	Required for flat plate		
Total					\$502.57			

Table 3: Bill of Materials



Plume-Surface Interactions for the Moon and Mars

2.11. Gantt chart





2.12. Budget

The preliminary budget provided by the Mechanical Engineering department for this project was \$750. This amount will be negotiable over time as the project evolves and more materials are required.

2.13. Deliverables at the end of the semester

The group aims to have a flat plate which will be able to change angles as well be sturdy enough to accurately gather data. The nozzle will also be able to effectively aim Mach 1 flow onto the flat plate. The group also aims to have results that will be accurately backed up by CFD models.

Conclusion

3.1. Results and Discussion

Overall, there is much to improve from the pre-existing sonic impingement design. The team is confident in their choice to implement robotic actuators in their alpha prototype. Further technical analysis will reveal the feasibility, pros, and cons to each concept design. As the team approaches fabrication, additional considerations are the feasibility of each design to work in a vacuum, the current/voltage draw of each design, and the magnitude and effect of vibrations caused by the start of the sonic jet.

3.2. Lessons learned and recommendations

The team was introduced to the wind tunnel where they learned how speeds over Mach 1 were established and could be controlled to be faster or slower. The team was also introduced to Schlieren imaging which can be used to see the flow that is established on the testing object. The Schlieren imaging will be useful when experimenting with the flat plate as it will allow for visual comparisons between different tests, angles, and speeds.



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Appendix A:

$$P_t = P_s \left(1 + \left(\frac{\gamma - 1}{2} \right) M^2 \right)^{\left(\frac{\gamma}{\gamma - 1} \right)} \tag{1}$$

$$P_t = P_s \left(\frac{\gamma + 1}{2}\right)^{\left(\frac{\gamma}{\gamma - 1}\right)} \tag{2}$$

where

• $P_t = \text{Stagnation Pressure}$

- $\gamma =$ Specific Heat Ratio
- ullet $P_s = \text{Static Pressure within the Reservoir}$
- M = Mach Number